Journal of Mathematical Sciences: Advances and Applications
Volume 2, Number 2, 2009, Pages 161-183

AN UNCERTAIN REASONING APPROACH WITH APPLICATIONS TO IMAGE RECOGNITION

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Abstract

In order to be able to better deal with uncertain information, this paper presents an uncertain reasoning approach based on rough set theory and other uncertainty theories. This paper studies mainly the application of the reasoning approach on image recognition. The simulation results show, the recognition precision based on the new reasoning approach is improved from previous 75.95 percent to now 83.33 percent at average, and the new reasoning approach has also some advantages, such as, it has the faster recognition speed, the lower storage capacity, and does not need any prior

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2000 Mathematics Subject Classification: TP311.

Keywords and phrases: rough sets, basic degree of belief, threshold value, image recognition.

This work is supported by Doctor fund of ZZULI.

Submitted by Jianxin Dai.

Received November 14, 2008

information in addition to data processing, these results indicate that the reasoning approach is more effective and feasible than the old reasoning approaches. Moreover, this paper also makes a comprehensive comparison to the new reasoning approach and the old reasoning approaches. Finally, some prospects for future research are given. In this paper, these researches on the reasoning approach for the image recognition not only are of important theoretical value to image processing, but also promote the applications of navigation systems and target recognition.

1. Introduction

In multi-sensor information fusion system, the information provided by each sensor is generally incomplete, imprecise, vague, and might even be contradictory, which includes a lot of uncertainty. The information fusion center had to rely on the uncertainty information to perform the reasoning in order to attain the purposes of the target identification and attributes judgment. In fact, the uncertain reasoning is a base for target identification and attributes information fusion. The intelligence in artificial intelligence systems mainly reflects the capabilities in solving uncertainty problems. Therefore, the approximate reasoning is a core research problem of artificial intelligence and expert system.

In previous works [1, 12-16, 18, 19], some uncertain inference approaches, such as subjective Bayes method [16], theory of evidence [14, 18, 19, 21], fuzzy inference [12], etc., have been successfully applied in artificial intelligence and expert system. But these theories all need the priori information outside the data dealt with, such as, Bayesian method requires all probabilities are independent, at the same time, we also need to know the priori probability and the conditional probability. When the Dempster rule of combination in theory of evidence is used, the Dempster rule of combination requires the evidence must be independent. However, the rough set [2-11, 17, 20, 22, 23] is a new mathematical theory to deal with imprecise, incomplete and uncertain data. The theory compares with other theories to deal with the uncertain and imprecise problems, the most notable difference is that it does not need to provide any prior information outside data processing, so it is more objective for the description or treatment of uncertainty of problem. Because the rough sets theory did not contain a mechanism to deal with imprecise or uncertain raw data, there are strongly complementary properties each other between the theory and other theories, such as probability theory, fuzzy theory, and theory of evidence to deal with uncertain or imprecise problems. Thus, this paper mainly studies the combinative reasoning approach of these theories and its applications, so far, which this combinative reasoning approach has never been studied all along.

For studying a new uncertain reasoning approach, this paper first introduces the basic concepts of rough set theory of the set-valued function, discusses the basic algorithm of the new uncertain reasoning approach, and gives the method of decision-making. At the same time, we provide example and simulation. By the example and simulation, the new reasoning approach compares with other imprecise reasoning approaches [11, 13], which it is not only more effective than the old reasoning approaches to image recognition, but also has the faster processing speed, lower storage capacity and communications traffic. In addition, we give the comparison of the new reasoning approach and other imprecise reasoning approaches. These researches and application of the reasoning approach in this paper not only develop the theory of imprecise reasoning, but also extend the application of the imprecise reasoning in various uncertainties information processing, for example, in artificial intelligence, expert systems, pattern recognition and image processing, etc.

2. Rough Set Theory

2.1. Basic knowledge of rough set theory

For studying a new reasoning approach by using the rough set (RS) theory, we must first know the basic theory [10, 17, 21] of RS.

Definition 2.1. Let Ω be a nonempty finite universe of discourse, and a set-valued mapping $R : \Omega \to 2^{\Omega}$ be a indistinguishable duality equivalence relation on Ω, i.e., the equivalence relation is an attribute. Where 2^{Ω} denotes the set that consists of all subsets of Ω. The ordered pair $\mathbb{K} = (\Omega, R)$ is an approximation space. For $\forall X \in 2^{\Omega}$, the lower approximation X_L and the upper approximation X_U of X regarding $\mathbb{K}$ are defined, respectively:

$$X_L = \{u \in \Omega | R(u) \subseteq X\},$$

$$X_U = \{u \in \Omega | R(u) \bigcap X \neq \emptyset\}.$$

When $X_L = X_U$, X regarding the approximation space $\mathbb{K}$ is called to be *definable* or *exact*. Otherwise X is called to be indefinable or rough, and it is called the rough set (RS), label $X = \langle X_L, X_U \rangle$.

The set $\mathrm{pos}(X) = X_L$ is called the positive region of X regarding $\mathbb{K}$, which it is the biggest definable set that consists of the members judged affirmatively belonging to X on the basis of existing knowledge R. The upper approximation X_U is the smallest definable set that consists of the members judged possibly belonging to X on the basis of existing knowledge R. $\mathrm{neg}(X) = \Omega - X_U$ is called a negative region of X regarding $\mathbb{K}$, which is a set that consists of the members judged affirmatively not belonging to X on the basis of existing knowledge R. The set $\mathrm{bn}(X) = X_U - X_L$ is called the boundary of X, it is a set that consists of the members judged possibly belonging to X but not entirely sure whether affirmatively belonging to X on the basis of existing knowledge R.

Definition 2.2. For the set-valued mapping function $R : \Omega \to 2^{\Omega}$, define the relation partition function $j : 2^{\Omega} \to 2^{\Omega}$ as follows:

$$j(X) = \{x \in \Omega | R(x) = X\}, \quad X \in 2^{\Omega}$$

$\mathbb{C} = \{X \in 2^{\Omega} | j(X) \neq \emptyset\}$ is called the *core set* of X.

Theorem 2.1. *The relation partition function j satisfies the following properties*:

(j1) $\bigcup_{X \subseteq \Omega} j(X) = \Omega$,

(j2) $X \neq Y,\ X, Y \subseteq \Omega \Rightarrow j(X) \bigcap j(Y) = \emptyset$.

Proof. (j1) By Definition 2.2, $\bigcup_{X \subseteq \Omega} j(X) = \bigcup_{X \subseteq \Omega} \{x \in \Omega | R(x) = X\} = \Omega$ is obvious.

(j2) Since R is a mapping, for $X, Y \subseteq \Omega$ (i.e., $X, Y \in 2^{\Omega}$), there are the corresponding original image $x \in \Omega$ and $y \in \Omega$ so as to $X = R(x)$ and $Y = R(y)$, respectively. When $X \neq Y$, there must be $x \neq y$. By Definition 2.2, then $j(X) \bigcap j(Y) = \emptyset$.

Assume $\mathbb{D} = \{j(X) | X \in \mathbb{C}\}$. Based on the Theorem 2.1, the set $\mathbb{D}$ forms a partition of Ω.

By Definitions 2.1 and 2.2, there is the following theorem [7].

Theorem 2.2. *Let the set-valued mapping* $R : \Omega \to 2^{\Omega}$ *be an equivalence relation on* Ω. *Assume* $j : 2^{\Omega} \to 2^{\Omega}$ *is a relation partition function. Then, for* $X \in 2^{\Omega}$, *the approximate operators and relation partition function have the following relation*:

(1) $X_L = \bigcup_{Y \subseteq X} j(Y)$,

(2) $X_U = \bigcup_{Y \bigcap X \neq \emptyset} j(Y)$,

where $Y = R(u)$, $u \in \Omega$.

2.2. Knowledge representation in RS theory

The knowledge representation mode in RS theory is generally expressed as the information table or information systems, i.e., it can be expressed as a quaternary ordered group $\mathbb{S} = (\Omega, R, V, \rho)$, where Ω is a set of objects, i.e., universe of discourse; R is a set of attributes; $V = \bigcup_{a \in R} V_a$, V_a is a range of the attribute a; $\rho : \Omega \times R \to V$ is an information function, which it endows with an information value to each attribute of every object, i.e., $\forall a \in R$, $x \in \Omega$, $\rho(x, a) \in V_a$.

In information systems $\mathbb{S} = (\Omega, R, V, \rho)$, sometimes the attribute set R is also divided into condition attribute C and decision attribute D, then the information systems is called the *decision table*, usually label $(\Omega, C \bigcup D, V, \rho)$.

In the decision table of RS, we can give the expression of decision rule by

$$r : \bigwedge(c, v) \to \bigvee(d, w),$$

where $c \in C, d \in D, v \in V_c, w \in V_d$. $\bigwedge(c, v)$ is called the *conditions part* of the rule and $\bigvee(d, w)$ is called the *decision-making part* of the rule.

3. Semantic Reasoning Approach

3.1. Elementary knowledge descriptions

In this paper, the reasoning approach is mainly based on the image semantic recognition, so the new reasoning approach is called the *semantic reasoning*.

(1) Basic Degree of Belief

Here we introduce the basic degree of belief is similar to the basic probability distribution function in theory of evidence [14, 18, 19].

Definition 3.1. Let Ω be a universe of discourse and X be a RS on Ω. *A* set function $m : 2^{\Omega} \to [0, 1]$ is called a *basic degree of belief* or a *basic belief function*, if it satisfies:

(m1) $m(\emptyset) = 0$,

(m2) $\sum_{X \subseteq \Omega} m(X) = 1$.

According to $\emptyset_L = \emptyset_U = \emptyset, \Omega_L = \Omega_U = \Omega$. Assume $m_1(X_L)$ and $m_2(X_U)$ are the basic belief function, i.e., the two all satisfy (m1) and (m2) of Definition 3.1. Then $m(X)$ can be computed by $m_1(X_L)$ and $m_2(X_U)$, i.e.,

$$\min\{m_1(X_L),\ m_2(X_U)\} \leqslant m(X) \leqslant \max\{m_1(X_L),\ m_2(X_U)\}. \tag{1}$$

Finally, $m(X)$ is given concretely by experts based on the formula (1).

Although experts can endow with any degree of belief for an event X to make it satisfy the formula (1), it requires the total sum to the degree of belief for all events is 1. However, to the computation of $m_1(X_L)$ and $m_2(X_U)$, we may refer to the calculation of the basic probability distribution function in theory of evidence [18].

In addition, if a lot of experts (for example, n experts) evaluate to the same event X, the basic degree of belief $m(X)$ is calculated by the following formula:

$$m(X) = \sum_{i=1}^{n} w_i m_i(X), \tag{2}$$

where $m_i(X)$ is the degree of belief that the i-th expert endows with the event X, w_i is the weights that the i-th expert gives, and $0 \leqslant w_i \leqslant 1$. According to people's practice to the uncertainty information processing, w_i is usually calculated by the normal membership function.

Definition 3.2. Assume $\mathbb{K} = (\Omega, R)$ is an approximation space, $X \subseteq \Omega$ is a RS on $\mathbb{K}$, and $m : 2^{\Omega} \to [0, 1]$ is a basic degree of belief on Ω. Define a function $B^* : 2^{\Omega} \to [0, 1]$ is:

$$B^*(X) = \sum_{D \subseteq X} m(D), \quad \forall X \subseteq \Omega. \tag{3}$$

The function is called a belief function on Ω, where $D = R(u)$, $u \in \Omega$.

By Definitions 3.1 and 3.2, obviously, there are $B^*(\emptyset) = 0$ and $B^*(\Omega) = 1$.

Definition 3.3. Assume $\mathbb{K} = (\Omega, R)$ is an approximation space, $X \subseteq \Omega$ is a RS on $\mathbb{K}$, and m is a basic degree of belief on Ω. Define a function $P^* : 2^{\Omega} \to [0, 1]$ is:

$$P^*(X) = 1 - B^*(\neg X) = \sum_{D \bigcap X \neq \emptyset} m(D), \quad X \subseteq \Omega. \tag{4}$$

The function P^* is called a plausibility function on Ω, where $D = R(u)$, $u \in \Omega$, $\neg$ is a complementary set of X.

By Definitions, 3.2 and 3.3, there is $B^*(X) \leqslant P^*(X)$.

According to the basic degree of belief, we can obtain the belief function and the plausibility function. Conversely, based on the known belief function, the basic degree of belief can be obtained by the Möbius transformation of B^* :

$$m(X) = \sum_{D \subseteq X} (-1)^{|X - D|} B^*(D), \quad X \subseteq \Omega, \tag{5}$$

where $D = R(u), \quad u \in \Omega$.

Regarding the proof of formula (5), refer to [17].

(2) Relationship of Degree of Belief and Approximation Set

The relationship among the basic degree of belief, the probability of approximation operation of RS, X and the probability of equivalence class can be described by the following theorem.

Theorem 3.1. *Assume* (Ω, R) *is an approximation space*, Pr *is probability measure on the subset of* Ω, *the triple* $(\Omega, R, \Pr)$ *is an approximate probability space*, *m is the basic degree of belief on* Ω. *To the set-valued function* $R : \Omega \to 2^{\Omega}$, *if there is* $R(u) \neq \emptyset$ *for any* $u \in \Omega$, *then*

(P1) $m(X) = \Pr(j(X)), X \in 2^{\Omega}$,

(P2) $B^*(X) = \Pr(X_L), X \in 2^{\Omega}$,

(P3) $P^*(X) = \Pr(X_U), X \in 2^{\Omega}$.

Proof. (P1) Firstly, since there is $R(u) \neq \emptyset$ for any $u \in \Omega$, $j(\emptyset) = \emptyset$ by Definition 2.2. Then $\Pr(j(\emptyset)) = \Pr(\emptyset) = 0$.

Secondly, according to (j2) and (j1) of Theorem 2.1, there is

$$\sum_{X \subseteq \Omega} \Pr(j(X)) = \Pr\left(\bigcup_{X \subseteq \Omega} j(X)\right) = \Pr(\Omega) = 1.$$

Also m is the basic degree of belief on Ω, $m(\emptyset) = 0$ and $\sum_{X \subseteq \Omega} m(X) = 1$ hold by Definition 3.1. According to the meaning of m and probability again, so $m(X) = \Pr(j(X))$.

(P2) By Theorems 2.2 and 2.1, (P1) and Definition 3.2, there is

$$\Pr(X_L) = \Pr\left(\bigcup_{Y \subseteq X} j(Y)\right) = \sum_{Y \subseteq X} \Pr(j(Y)) = \sum_{Y \subseteq X} m(Y) = B^*(X), \text{ i.e., } B^*(X) = \Pr(X_L).$$

(P3) By Theorems 2.2 and 2.1, (P1) and Definition 3.3, there is

$$\Pr(X_U) = \Pr\left(\bigcup_{Y \bigcap X \neq \emptyset} j(Y)\right) = \sum_{Y \bigcap X \neq \emptyset} \Pr(j(Y)) = \sum_{Y \bigcap X \neq \emptyset} m(Y) = P^*(X),$$

where $X \in 2^{\Omega}$.

According to this theorem, some conclusions can be obtained as follows:

The basic degree of belief $m(X)$ of X is the probability of a subset of universe of discourse Ω. The belief function $B^*(X)$ and plausibility function $P^*(X)$ are the probability of lower approximation and the probability of upper approximation about RS X, respectively. Then $m(X)$ denotes the exact degree of belief to proposition (or events) X; $B^*(X)$ denotes the degree of belief to affirm the events X must be true, and is called the lower probability measure of X; $P^*(X)$ denotes the degree of belief to judge the events X is non-pseudo, i.e., the degree of belief to do non negative X, and is called the upper probability measure of X. Since $X_L \subseteq X_U$, then $\Pr(bn(X)) = \Pr(X_U - X_L) = \Pr(X_U) - \Pr(X_L) = P^*$

$(X) - B^*(X)$, so $P^*(X) - B^*(X)$ describes the uncertainty caused by the information that cannot be known to X, and is called the uncertainty of X. $[B^*(X), P^*(X)]$ is called the confidence interval of X, which describes the uncertainty of X. $\Pr(neg(X)) = \Pr(\Omega - X_U) = \Pr(\Omega) - \Pr(X_U) = 1 - P^*(X) = B^*(\neg X)$ denotes the degree of belief to affirm the events X must be pseudo, i.e., the degree of belief to negative X.

The relationship between the basic degree of belief and approximation operator of RS X is described as follows:

Define $\alpha = \min_{D \subseteq X}\{m(D)\}$, then $m(D) \geqslant \alpha$ for $\forall D \subseteq X$.

Define $\beta = \min_{D \bigcap X \neq \emptyset}\{m(D)\}$, then $m(D) \geqslant \beta$ for $\forall D \bigcap X \neq \emptyset$.

By the definition, there is $0 \leqslant \beta \leqslant \alpha \leqslant 1$.

For $0 \leqslant \beta \leqslant \alpha \leqslant 1$, by Definition 2.1, X_L and X_U can also be expressed as:

$$X_L = \{u \in \Omega \mid m(D) \geqslant \alpha,\ D = R(u)\}, \tag{6}$$

$$X_U = \{u \in \Omega \mid m(D) \geqslant \beta,\ D = R(u)\}. \tag{7}$$

(3) Calculation of Degree of Belief

Except the basic degree of belief is given by expert system and intelligent theory, it can be also obtained by computing the probability according to Theorem 3.1, such as it can be got by Bayesian methods [16]. However, when the priori probability is unknown, we will adopt the following method: if the experts give the basic degree of belief $m(E)$ of initial evidence E or the basic degree of belief $m(H)$ of conclusion H, the user can give the degree of belief $m(E|S)$ or $m(H|S)$ on the basis of observation S.

Here, solve $m(E|S)$ as an example. To convenient for users determine $m(E|S)$, the concept of credibility is introduced here. We use an integer among $-10 \sim 10$ as the credibility $C(E|S)$ of evidence, users can choose on the basis of actual situation. The corresponding relationship between the credibility $C(E|S)$ and the basic degree of belief $m(E|S)$ is described as follows:

(1) $C(E|S) = -10$ denotes the evidence E must be inexistence under observation S, i.e., $m(E|S) = 0$;

(2) $C(E|S) = 0$ shows S has nothing to do with E, i.e., $m(E|S) = m(E)$;

(3) $C(E|S) = 10$ denotes the evidence E must be existence under observation S, i.e., $m(E|S) = 1$;

(4) when $C(E|S)$ is other values, the corresponding relationship of $C(E|S)$ and $m(E|S)$ can be obtained by carrying out piecewise linear interpolation to the above three points, i.e.,

$$m(E|S) = \begin{cases} \dfrac{C(E|S) + m(E)[10 - C(E|S)]}{10}, & 0 \leqslant C(E|S) \leqslant 10 \\ \dfrac{m(E)[10 + C(E|S)]}{10}, & -10 \leqslant C(E|S) \leqslant 0. \end{cases} \tag{8}$$

In this way, $C(E|S)$ can be converted to the corresponding $m(E|S)$ by the system as long as users give the credibility $C(E|S)$ to the initial evidence. Similarly, $m(H|S)$ can be obtained.

(4) Combination of Evidence

Here, the combination of evidence discussed denotes the synthesis of degree of belief.

(1) Using Dempster combination rule [14, 18, 19]

When the evidences are independent each other to derive the same conclusion, we use the Dempster combination rule to perform the combination of evidence.

Assume the universe of discourse Ω is a recognition framework, m_1 and m_2 are the basic degree of belief that are independent of each other on 2^{Ω}. For any $X_i, Y_j \subseteq \Omega$ and $X_i, Y_j \neq \emptyset$, calculate

$$N = \sum_{i,\,j,\,X_i \bigcap Y_j \neq \emptyset} m_1(X_i)m_2(Y_j) > 0 \quad \text{or}$$

$$K = \sum_{i,\,j,\,X_i \bigcap Y_j = \emptyset} m_1(X_i)m_2(Y_j) < 1, \quad N = 1 - K. \tag{9}$$

Then $m = m_1 \oplus m_2$ is

$$m(Z) = \begin{cases} \dfrac{\sum\limits_{i,\,j,\,X_i \bigcap Y_j = Z} m_1(X_i)m_2(Y_j)}{N}, \forall Z \subseteq \Omega,\ Z \neq \emptyset \\ 0,\ Z = \emptyset. \end{cases} \tag{10}$$

On formula (10), if $K \neq 1$, m is confirmed to be a basic degree of belief, if $K = 1$, m_1 and m_2 are thought to be contradictive, and, then the basic degree of belief cannot be combined. In addition, for the combination of more evidence, we perform pairwise combination to evidence.

(2) Fuzzy combination

When the evidences are not independent to derive the same conclusion, we use the operation rules of fuzzy set or fuzzy synthetic function to perform the combination of evidence. Here we give the operation rule of fuzzy synthetic function.

Assume the universe of discourse Ω is a recognition framework, m_i $(i = 1, 2, \cdots, n)$ are the basic degree of belief on 2^{Ω}. For $X \subseteq \Omega$, assume $M_n(X) = (m_1(X), m_2(X), \cdots, m_n(X))'$, where $M_n(X) \in [0, 1]^n$.

Usually take the fuzzy synthetic function $\mathbb{S}_n$ is

$$\mathbb{S}_n(M_n(X)) = \left(\frac{1}{n}\sum_{i=1}^{n} m_i^q(X)\right)^{\frac{1}{q}}, \ q > 0. \tag{11}$$

3.2. Semantic reasoning

After the evidences are combined by the above combination methods, how to perform decision-making is closely related to the application.

Definition 3.4. Assume the universe of discourse Ω is a recognition framework, m is the combined basic degree of belief based on the above combination rules. A decision-making project is called a *semantic reasoning*, if the decision-making project satisfies the following one of decision-making methods:

(1) In accordance with the combined basic degree of belief m, the belief function B^* is obtained, as is our decision-making result.

(2) According to the attribute reduction theory of RS, further reduce the range of the true values, so as to obtain the decision-making result at last. This method is: for RS $X \subseteq \Omega$, the belief function is $B^*(X)$, if the new set Y_1 that we remove a certain element from the set X is obtained and its belief function is $B^*(Y_1)$, at the same time $|B^*(X) - B^*(Y_1)| < \varepsilon$, then we think the element can be removed from X, where ε is a predetermined threshold value. Repeat this process until the subset Y_K has not the element can be removed from it, i.e., until Y_K is the smallest reduction, then Y_K is the decision-making result.

(3) Assume $\exists X_1, X_2 \subseteq \Omega$ satisfies

$$m(X_1) = \max\{m(X_i), X_i \subseteq \Omega\} \tag{12}$$

$$m(X_2) = \max\{m(X_i), X_i \subseteq \Omega, \text{ but } X_i \neq X_1\}. \tag{13}$$

If there is:

$$\begin{cases} m(X_1) - m(X_2) > \varepsilon_1 \\ m(\Omega) < \varepsilon_2 \\ m(X_1) > m(\Omega), \end{cases} \tag{14}$$

then X_1 is the decision-making result, where ε_1 and ε_2 are a predetermined threshold value, respectively.

(4) Assume m is the degree of belief on Ω. For $X \subseteq \Omega$, compute the approximate precision $\mu = B^*(X)/P^*(X)$. We determine the decision-making result by solving $m_{\max}$ and $\mu_{\max}$.

4. Application of Semantic Reasoning and Comparison with Other Uncertainty Reasoning Approaches

4.1. Application of semantic reasoning on semantic recognition of image

To illustrate the application of semantic reasoning, an example on semantic recognition of image here is given through information data fusion.

Example 4.1. Assume an image X consists of three targets, which indicate three different aircrafts. The three aircrafts may include battleplane o_1, multi-purpose ground attack airplane o_2, bomber o_3, early warning airplane o_4 and other aircraft o_5, then the universe of discourse $\Omega = \{o_1, o_2, o_3, o_4, o_5\}$ is a target recognition framework, as is shown in Figure 1. Assume R is an indistinguishable equivalence relation on Ω, through the equivalence relation R, its equivalence class can be obtained as follows:

$$E_1 = \{o_1\}, \quad E_2 = \{o_2\}, \quad E_3 = \{o_3, o_4\}, \quad E_4 = \{o_5\}. \tag{15}$$

The image $X \subseteq \Omega$ has three targets, and it must include one of o_3 and o_4. The targets can be identified by measuring some attribute values of target image. Here, the used attributes are the radio frequency (*RF*), pulse width (*PW*), wavelength (*WL*) and speed (*SP*). The system uses three different categories sensor to measure these attribute values so as to make sure these attribute values are the basic degrees of belief of a certain target, as is shown in Table 1. Try to solve the following question: the total degree of belief that the target must be in X, i.e., the degree of belief of positive region of X; confidence interval; the total degree of belief that the target is certainly not in X, i.e., the degree of belief of negative region of X; the uncertainty caused by the information that cannot be known to X, i.e., the degree of belief of boundary of X. So the recognition to X can be attained.

Figure 1. Different aircraft for operation.

Table 1. Basic degree of belief determined by sensors

Basic degree of belief		Attributes			
		RF	*PW*	*WL*	*SP*
Universe of discourse Ω	o_1	0.20	0.45	0.25	0.40
	o_2	0.40	0.05	0.30	0.40
	o_3	0.12	0.25	0.00	0.00
	o_4	0.15	0.10	0.20	0.00
	o_5	0.00	0.00	0.00	0.00
	Ω	0.13	0.15	0.25	0.20

Solution. In order to achieve the semantic recognition of the image, each target in the image must correctly be identified.

Make sure the degree that the target belongs to X by fusion of various attributes, which is shown as follows:

After the fusion of attributes *RF* and *PW* is carried out, the degree of belief to the target is the synthesis of $m_{RF}(\cdot)$ and $m_{PW}(\cdot)$, as is shown in Table 2, and in Table 2, $\emptyset$ denotes the empty set. Based on Table 2 and formula (10), the inconsistency factor K for $m_{RF}(\cdot)$ and $m_{PW}(\cdot)$ is $K = 0.5845$.

Therefore, according to attributes *RF* and *PW*, the basic degrees of belief to target identification are, respectively

$$m_{RF\times PW}(o_1) = \frac{0.09 + 0.03 + 0.0585}{1 - K} \approx 0.43,$$

$$m_{RF\times PW}(o_2) = \frac{0.02 + 0.06 + 0.0065}{1 - K} \approx 0.21,$$

$$m_{RF\times PW}(o_3) = \frac{0.03 + 0.018 + 0.0325}{1 - K} \approx 0.19,$$

$$m_{RF\times PW}(o_4) = \frac{0.015 + 0.0225 + 0.013}{1 - K} \approx 0.12,$$

$$m_{RF\times PW}(o_5) \approx 0,$$

$$m_{RF\times PW}(\Omega) = \frac{0.0195}{1 - K} \approx 0.05.$$

Similarly, after the fusion of attributes $RF \times PW$ and *WL* is carried out, the basic degrees of belief to target identification are, respectively

Table 2. Synthesis of $m_{RF}(\cdot)$ and $m_{PW}(\cdot)$

	$m_{RF}(\cdot)$					
	$o_1(0.09)$	$\emptyset(0.18)$	$\emptyset(0.054)$	$\emptyset(0.0675)$	$\emptyset(0)$	$o_1(0.0585)$
	$\emptyset(0.01)$	$o_2(0.02)$	$\emptyset(0.006)$	$\emptyset(0.0075)$	$\emptyset(0)$	$o_2(0.0065)$
$m_{PW}(\cdot)$	$\emptyset(0.05)$	$\emptyset(0.10)$	$o_3(0.03)$	$\emptyset(0.0375)$	$\emptyset(0)$	$o_3(0.0325)$
	$\emptyset(0.02)$	$\emptyset(0.04)$	$\emptyset(0.012)$	$o_4(0.0150)$	$\emptyset(0)$	$o_4(0.0130)$
	$\emptyset(0)$	$\emptyset(0)$	$\emptyset(0)$	$\emptyset(0)$	$o_5(0)$	$o_5(0)$
	$o_1(0.03)$	$o_2(0.06)$	$o_3(0.018)$	$o_4(0.0225)$	$o_5(0)$	$\Omega(0.0195)$

$m_{RF\times PW\times WL}(o_1) = 0.480$, $m_{RF\times PW\times WL}(o_2) = 0.270$, $m_{RF\times PW\times WL}(o_3) = 0.100$,

$m_{RF\times PW\times WL}(o_4) = 0.133$, $m_{RF\times PW\times WL}(o_5) = 0$, $m_{RF\times PW\times WL}(\Omega) = 0.027$.

After the fusion of attributes $RF \times PW \times WL$ and SP is carried out, and assume $\text{all} = RF \times PW \times WL \times SP$, the basic degrees of belief to target identification are, respectively

$$m_{\text{all}}(o_1) = 0.58, \quad m_{\text{all}}(o_2) = 0.33, \quad m_{\text{all}}(o_3) = 0.03,$$

$$m_{\text{all}}(o_4) = 0.05, \quad m_{\text{all}}(o_5) = 0, \quad m_{\text{all}}(\Omega) = 0.01.$$

According to the semantic reasoning, when use the decision-making method 3, if we choose the threshold value $\varepsilon_1 = \varepsilon_2 = 0.1$, the first decision-making result is o_1, i.e., confirm X has a target is battleplane. Secondly, remove o_1 from Ω, and use the same decision-making method to other elements in Ω, then the decision-making result is o_2. Repeat this process until this decision-making method cannot be used again, then $X = \{o_1, o_2, *\}$, where $*$ is an uncertain target that is o_3 or o_4.

According to the equivalence class (15) of R, the lower approximation X_L and the upper approximation X_U of X can be obtained as follows:

$$X_L = E_1 \bigcup E_2 = \{o_1, o_2\}, \tag{16}$$

$$X_U = E_1 \bigcup E_2 \bigcup E_3 = \{o_1, o_2, o_3, o_4\}. \tag{17}$$

Then the degree of belief that X must include o_1 and o_2 is

$$B^*(X) = 0.58 + 0.33 = 0.91.$$

By $m_{\text{all}}(\{o_3, o_4\}) = 0.03 \wedge 0.05 = 0.03$, the degree of belief that X may include all possible targets is

$$P^*(X) = 0.91 + 0.03 = 0.94.$$

The confidence interval of X is [0.91, 0.94], which describes the uncertainty of X.

The degree of belief of negative region of X is

$$B^*(\neg X) = 1 - P^*(X) = 0.06.$$

The degree of belief of boundary of X, i.e., the uncertainty caused by the information that cannot be known to X is

$$m(\text{bn}(X)) = P^*(X) - B^*(X) = 0.94 - 0.91 = 0.03. \tag{18}$$

According to the formula (16) and (17), we now show some degrees of belief before fusion of attribute, i.e., the degree of belief that a single attribute RF to determine that X is true is $B_0^*(X) = 0.60;$ the degree of belief that X is not pseudo is $P_0^*(X) = 0.72;$ the uncertainty of X is

$$m_0(\text{bn}(X)) = P_0^*(X) - B_0^*(X) = 0.12. \tag{19}$$

From $B_0^*(X)$ and $B^*(X)$, formula (18) and (19) seen, through fusion of attributes, the degree of belief to confirm X must be true is improved to 0.91. The uncertainty caused by the information that cannot be known to X is dropped to 0.03. The computer simulation results also demonstrate the same conclusion, as is shown in Figure 2.

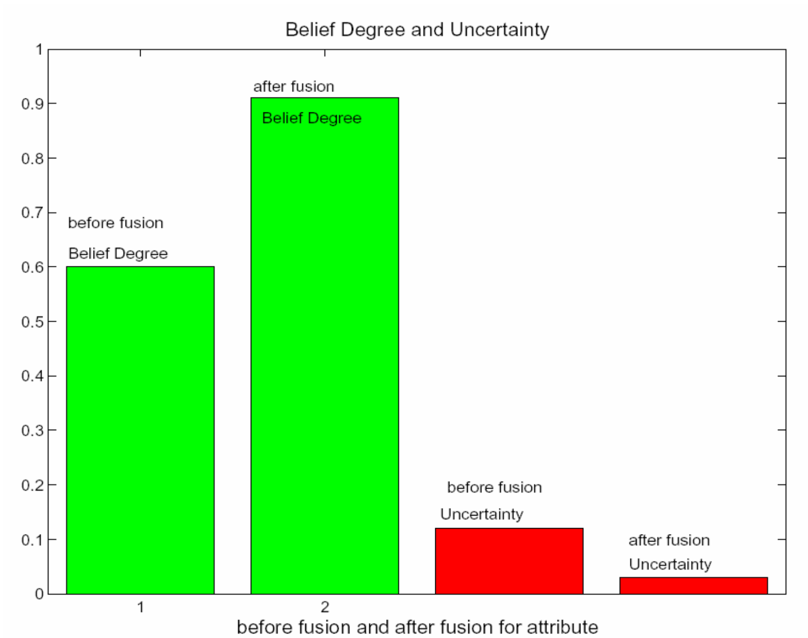

Figure 2. Belief degree and uncertainty before fusion and after fusion for attribute.

Compare the semantic reasoning with old reasoning [11, 13] to target recognition in dense target environment, and the recognition results are shown in Figure 3.

From Figure 3 known, the correct average recognition rates of old reasoning approaches [11, 13] are 67.55% and 72.35%, respectively, however, that of semantic reasoning approach is 80.8%, which shows the semantic reasoning approach is more effective than old reasoning approaches to target identification.

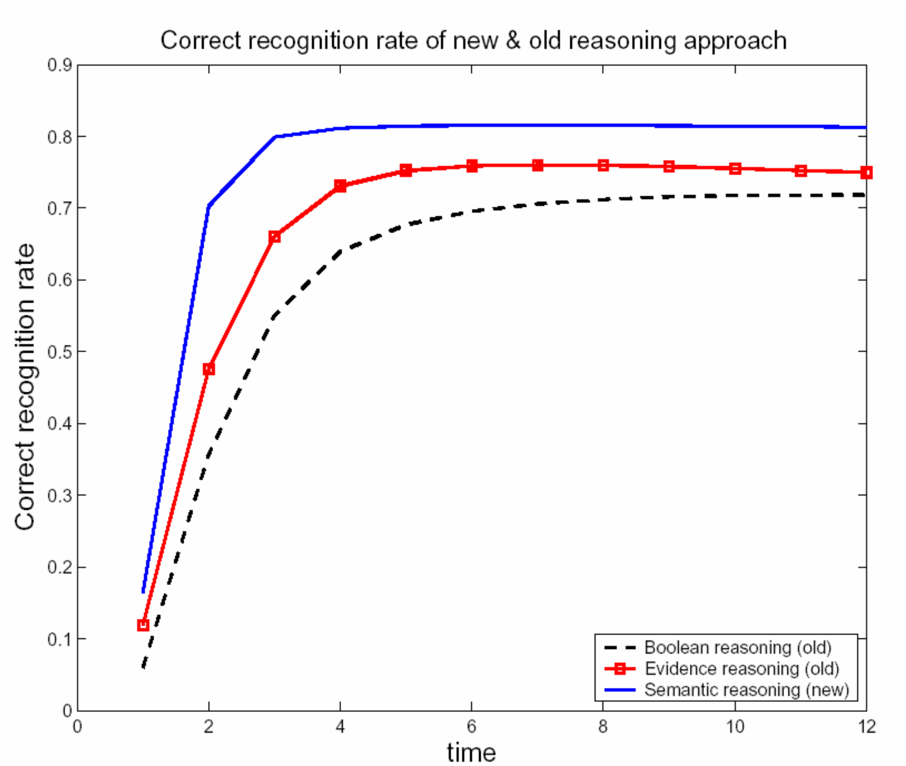

Figure 3. Comparison of semantic reasoning and old reasoning approaches to target identification.

If we further understand the meaning of the image X, we can continue observing to obtain the decision-making Table 3.

The decision attribute can be expressed by the degree of belief (WGC) to war game, firstly, the experts

Table 3. Decision-making Table

Image X	Condition Attributes			Decision Attribute (WGC)
	Throw Bombs	Exchange signal wave	Damaged facilities	
o_1	Yes	Yes	No	0.85
o_2	Yes	No	No	0.75
o_3	No	Yes	Yes	0.2
o_4	No	Yes	Yes	0.5

give the prior degree of belief $m(E)$ of war game, then the users calculate the degree of belief $m(E|S)$ under observation S (i.e., condition attribute values) according to formula (8), as shown in Table 3.

We take $\alpha = 0.65, \beta = 0.6.$ Define the lower approximation of war game (*WG*), i.e., the positive region is

$WG_L = pos(WG) = \{X$ must be the image of war game $|m_{WG}(X) > \alpha\}.$ Define the upper approximation of *WG* is

$WG_U = \{X$ is likely to be the image of war game $|m_{WG}(X) > \beta\}.$ Define the negative region of *WG* is

$neg(WG) = \{X$ is definitely not an image of war game $|m_{WG}(X) < \beta\}.$ Define the boundary of *WG* is

$bn(WG) = \{X$ may or may not be a war game $|\beta \leqslant m_{WG}(X) \leqslant \alpha\}.$

By fuzzy synthetic function formula (11), take $q = 1,$ we obtain the following: if $X = \{o_1, o_2, o_3\},$ then $m_{WG}(X) = 0.6 \geqslant \beta$ but $0.6 < \alpha,$ thus X may be a war game but is uncertain; If $X = \{o_1, o_2, o_4\},$ then $m_{WG}(X) = 0.7 > \alpha,$ thus we can affirm X is a war game.

4.2. Comparison of semantic reasoning and other uncertainty reasoning approaches

The comparison between it and other uncertainty reasoning approaches is given as follows:

(1) From the point of view to dealing with the uncertainty, the subjective Bayes method uses the probability to express the uncertainty, the evidence reasoning uses the degree of belief to indicate the uncertainty, however the semantic reasoning not only can use the probability, but also can use the degree of belief, as well as the probability of positive region, negative region and boundary to express the uncertainty.

(2) From the complexity of the calculation, the subjective Bayes method has the complexity of index information, the evidence reasoning has the complexity of index information and index time, however the semantic reasoning can decrease the computing time, and reduce the storage capacity by the reduction of knowledge (or attributes).

(3) The subjective Bayes method can not distinguish between uncertainty and ignorance, the evidence reasoning and the semantic reasoning all are able to distinguish between the two, however, the semantic reasoning is more intuitionistic to distinguish between the two by using positive region, negative region and boundary.

(4) The subjective Bayes method requires assuming the priori probability and conditional probability, the evidence reasoning requires the evidences are independent while using Dempster combination rule, however the semantic reasoning does not need to provide any prior information in addition to data processing.

(5) From the Example 4.1 known, the semantic reasoning requires setting the system parameters, however, the subjective Bayes method and the evidence reasoning do not need that.

5. Conclusions

According to RS theory, subjective Bayes method and the theory of evidence, this paper studies a new uncertain reasoning approach, and discusses its reasoning process. But this paper studies mainly the applications of the new reasoning approach in image recognition, and gives example and simulation. At the same time, we also make a comparison to the new reasoning approach and other uncertainty reasoning approaches.

In the semantic reasoning, how to construct the basic degree of belief based on the actual situation, set up the system parameters, and how to better combine expert system, these issues are important in practical application and will need to be studied. If the semantic reasoning can combine with other theories in practice, it will be more and more in-depth in the applications of the field of artificial intelligence, in particular in expert system and pattern recognition.

Acknowledgements

This work is supported by Nation Natural Science Fund Co-proposal (No: 90612014) and Nation 863 project (No: 2006AA01Z101).

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